

# Cosmic Equation of State, Quintessence and Decaying Dark Matter

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## Abstract

If CDM particles decay and their lifetime is comparable to the age of the Universe, they can modify its equation of state. By comparing the results of numerical simulations with high redshift SN-Ia observations we show that this Hypothesis is consistent with present data. Fitting the simplest quintessence models with constant  $w_q$  to data leads to  $w_q \lesssim -1$ . We show that a universe with a cosmological constant or quintessence matter with  $w_q \sim -1$  and a decaying Dark Matter has an effective  $w_q < -1$  and fits SN data better than stable CDM or quintessence models with  $w_q > -1$ .

There are at least two motivations for existence of a Decaying Dark Matter (DDM). If R-symmetry in SUSY models is not strictly conserved, the LSP which is one of the best candidates of DM can decay to Standard Model particles [1]. Violation of this symmetry is one of the many ways for providing neutrinos with very small mass and large mixing angle.

Another motivation is the search for sources of Ultra High Energy Cosmic Rays (UHECRs)(see [2] for review of their detection and [3] and [4] respectively for conventional and exotic sources). In this case, DDM must be composed of ultra heavy particles with  $M_{DM} \sim 10^{22} - 10^{25} eV$ . In a recent work [5] we have shown that the lifetime of UHDM can be relatively short, i.e.  $\tau \sim 10 - 100\tau_0$  where  $\tau_0$  is the age of the Universe. Here we compare the prediction of this simulation for modification of Cosmic equation of State (CES) with observation of high redshift SN-Ia.

For details of the simulation we refer the reader to [5]. In summary, the decay of UHDM is assumed to be like the hadronization of two gluon jets. The decay remnants interact with cosmic backgrounds, notably CMB, IR, and relic neutrinos, lose their energy and leave a high energy background of stable species e.i.  $e^\pm$ ,  $p^\pm$ ,  $\nu$ ,  $\bar{\nu}$ , and  $\gamma$ . We solve the Einstein-Boltzmann equations to determine the energy spectrum of remnants. Results of [5] show that in a homogeneous universe, even the short lifetime mentioned above can not explain the observed flux of UHECRs. The clumping of DM in the Galactic Halo however limits the possible age/contribution. These parameters are degenerate and we can not separate them. For simplicity, we assume that CDM is entirely composed of DDM and limit the lifetime. Fig. 1 shows the evolution of energy density  $T^{00}(z) \equiv \rho(z)$  at low and medium redshifts in a flat universe with and without a cosmological constant. As expected, the effect of DDM is more significant in a matter dominated universe i.e. when  $\Lambda = 0$ . For a given cosmology, the lifetime of DDM is the only parameter that significantly affects the evolution of  $\rho$ . For the same lifetime, the difference between  $M_{DM} = 10^{12} eV$  and  $M_{DM} = 10^{24} eV$  cases is only  $\approx 0.4\%$ . Consequently, in the following we neglect the effect of mass. For the same cosmological model and initial conditions, if DM decays, matter density at  $z = 0$  is smaller than when it is stable because decay remnants remain highly relativistic even after losing part of their energy. Their density dilutes more rapidly with the expansion of the Universe than CDM and decreases the total matter density and Hubble constant. Consequently, relative contribution of cosmological constant increases. This process mimics a dark matter with negative pressure i.e. a quintessence matter [8] [9] (see [10] for recent review). The equation of state of this "matter" however has an exponent  $w < -1$  which is different from prediction of quintessence models with positive potential.

The most direct way for determination of cosmological densities and equation of state is the observation of SN-Ia's as standard candles. It is based on the measurement of apparent magnitude of the maximum of SNs lightcurve [6] [7] [11]. After correction for various observational and intrinsic variations like

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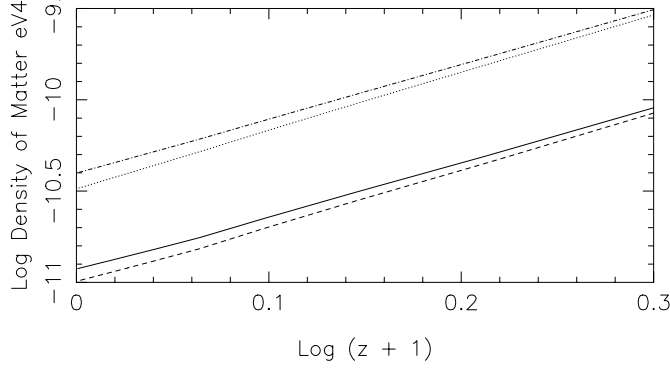


Figure 1: Energy density of the Universe. Solid line  $\Omega_{\Lambda}^{eq} = \Omega_{\Lambda} = 0.7$  and stable DM; dashed line the same cosmology with  $\tau = 5\tau_0$ ; dash dot line  $\Lambda = 0$  and stable DM; dot line  $\Lambda = 0$  and  $\tau = 5\tau_0$ . Dependence on the mass of DM is negligible.

K-correction, width-luminosity relation, reddening and Galactic extinction, it is assumed that their magnitude is universal. Therefore the difference in apparent magnitude is only related to difference in distance and consequently to cosmological parameters.

The apparent magnitude of an object  $m(z)$  is related to its absolute magnitude  $M$ :  $m(z) = M + 25 + 5 \log D_L$ , where  $D_L$  is the Hubble-constant-free luminosity distance:

$$D_L = \frac{(z+1)}{\sqrt{|\Omega_R|}} \mathcal{S}\{\sqrt{|\Omega_R|} \int_0^z \frac{dz'}{E(z')}\} \quad (1)$$

$$\mathcal{S}(x) = \begin{cases} \sinh(x) & \Omega_R > 0, \\ x & \Omega_R = 0, \\ \sin(x) & \Omega_R < 0. \end{cases} \quad (2)$$

$$E(z) = \frac{H(z)}{H_0}. \quad (3)$$

$$H^2(z) = \frac{8\pi G}{3} T^{00}(z) + \frac{\Lambda}{3}. \quad (4)$$

Here we only consider flat cosmologies.

We use the published results of the Supernova Cosmology Project [11] for high redshift and Calan-Tololo sample [12] for low redshift supernova and compare them our simulation. From these data sets we eliminate 4 SNs with largest residue and stretch as explained in [11] (i.e. we use objects used in their fit B).

Minimum- $\chi^2$  fit method is applied to data to extract parameters of cosmological models. In all fits described in this letter we consider  $M$  as a free parameter and minimize the  $\chi^2$  with respect to it. Its variation our in fits stays in the acceptable range of  $\pm 0.17$  [6].

We have restricted our calculation to a range of parameters close to the best fit of Perlmutter S. *et al.* [11] i.e.  $2.38 \times 10^{-11} \leq \rho_{\Lambda} \equiv \frac{\Lambda}{8\pi G} \leq 3.17 \times 10^{-11} eV^4$ . The reason why we use  $\rho_{\Lambda}$  rather than  $\Omega_{\Lambda}$  is that the latter quantity depends on the equation of state and the lifetime of the Dark Matter. The range of  $\rho_{\Lambda}$  given here is equivalent to  $0.6 \leq \Omega_{\Lambda}^{eq} \leq 0.8$  for a stable CDM and  $H_0 = 70 \text{ km Mpc}^{-1} \text{ sec}^{-1}$  (we use  $\Omega_{\Lambda}^{eq}$  notation to distinguish between this quantity and real  $\Omega_{\Lambda}$ ).

Fig.2 shows the residues of the best fit to DDM simulation. Although up to  $1\text{-}\sigma$  uncertainty all models with stable or decaying DM with  $5\tau_0 \lesssim \tau \lesssim 50\tau_0$  and  $0.68 \lesssim \Omega_{\Lambda}^{eq} \lesssim 0.72$  are compatible with data, a decaying DM with  $\tau \sim 5\tau_0$  systematically fits the data better than stable DM with the same  $\Omega_{\Lambda}^{eq}$ . Models with  $\Lambda = 0$  are ruled out with more than 99% confidence level.

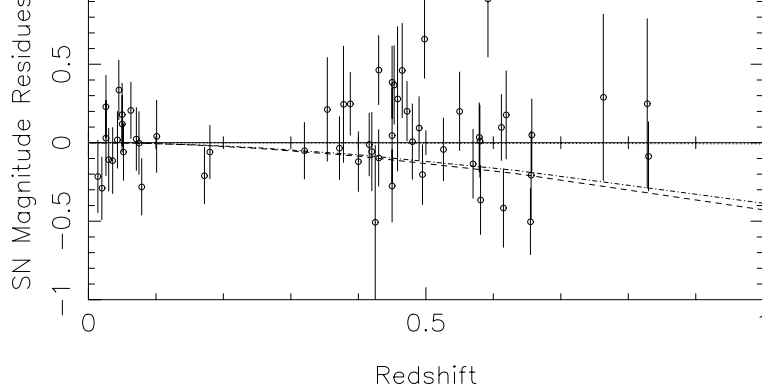


Figure 2: Best fit residues with  $\Omega_{\Lambda}^{eq} = 0.7$ ,  $\tau = 5\tau_0$ . It leads to  $\Omega_{\Lambda} = 0.73$ . The curves correspond to residue for stable DM with  $\Omega_{\Lambda}^{eq} = \Omega_{\Lambda} = 0.7$  (dotted);  $\Lambda = 0$  and  $\tau = 5\tau_0$  (dashed);  $\Lambda = 0$ , stable DM (dash-dot).

In fitting the results of DM decay simulation to data we have directly used equation (4) without defining any analytical form for the evolution of  $T^{00}(z)$ . It is not usually the way data is fitted to cosmological models [6] [7] [13]. Consequently, we have fitted an analytic model to the simulation for  $z < 1$  as it is the redshift range of the available data. It includes a stable DM and a quintessence matter. Its evolution equation is:

$$H^2(z) = \frac{8\pi G}{3}(T_{st}^{00} + \Omega_q(z+1)^{3(w_q+1)}). \quad (5)$$

The term  $T_{st}^{00}$  is obtained from our simulation when DM is stable. In addition to CDM, it includes a small contribution from hot components i.e CMB and relic neutrinos. For a given  $\Omega_{\Lambda}^{eq}$  and  $\tau$ , the quintessence term is fitted to  $T^{00} - T_{st}^{00} + \frac{\Lambda}{8\pi G}$ .<sup>2</sup> The results of this fit are  $\Omega_q$  and  $w_q$  which characterize an equivalent quintessence model for the corresponding DDM. The analytical model fits the simulation extremely good and the absolute value of relative residues is less than 0.2%. Results for models in the 1- $\sigma$  distance of the best fit is summarized in Table 1.

In the next step, we fit an analytical model to SN-Ia data. Its evolution equation is the following:

$$H^2(z) = \frac{8\pi G}{3}((1 - \Omega_q)(z+1)^3 + \Omega_q(z+1)^{3(w_q+1)}). \quad (6)$$

The aim for this exercise is to compare DDM equivalent quintessence models with data and with other models of the same type.

Fig.3 shows the  $\chi^2$  of these fits as a function of  $w_q$  for various values of  $\Omega_q$ . The reason behind using  $\chi^2$  rather than confidence level is that it directly shows the goodness-of-fit. As with available data all relevant models are compatible up to 1- $\sigma$ , the error analysis is less important than goodness-of-fit and its behavior in parameter-space.

Models presented in Fig.3 have the same  $\Omega_q$  as equivalent quintessence models obtained from DDM and listed in Tab.1. These latter models are shown too. In spite of statistical closeness of all fits, the systematic tendency of the minimum of  $\chi^2$  to  $w_q < -1$  when  $\Omega_q < 0.75$  is evident. The minimum of models with  $\Omega_q > 0.75$  has  $w_q > -1$ , but the fit is worse than former case. Between DDM models, one with  $\Omega_q = 0.71$  is very close to the best fit of (6) models with same  $\Omega_q$ . Regarding errors however,

<sup>2</sup>The exact equivalent model should have the same form as (6). However, it is easy to verify that in this case, the minimization of  $\chi^2$  has a trivial solution with  $w_q = -1$ ,  $\Omega_q = 0$ . Only one equation remains for non-trivial solutions and it depends on both  $w_q$  and  $\Omega_q$ . Consequently, there are infinite number of solutions.

The model we have used here generates a very good equivalent model to DDM with less than 2% error (Because CDM and quintessence terms are not fitted together,  $\Omega$  is not exactly 1).

Table 1: Cosmological parameters from simulation of a decaying DM and parameters of the equivalent quintessence model.  $H_0$  is in  $km\ Mpc^{-1}\ sec^{-1}$ .

	Stable DM			$\tau = 50\tau_0$			$\tau = 5\tau_0$		
	$\Omega_\Lambda^{eq} = 0.68$	$\Omega_\Lambda^{eq} = 0.7$	$\Omega_\Lambda^{eq} = 0.72$	$\Omega_\Lambda^{eq} = 0.68$	$\Omega_\Lambda^{eq} = 0.7$	$\Omega_\Lambda^{eq} = 0.72$	$\Omega_\Lambda^{eq} = 0.68$	$\Omega_\Lambda^{eq} = 0.7$	$\Omega_\Lambda^{eq} = 0.72$
$H_0$	69.953	69.951	69.949	69.779	69.789	69.801	68.301	68.415	68.550
$\Omega_\Lambda$	0.681	0.701	0.721	0.684	0.704	0.724	0.714	0.733	0.751
$\Omega_q$	-	-	-	0.679	0.700	0.720	0.667	0.689	0.711
$w_q$	-	-	-	-1.0066	-1.0060	-1.0055	-1.0732	-1.0658	-1.0590
$\chi^2$	62.36	62.23	62.21	62.34	62.22	62.21	62.22	62.15	62.20

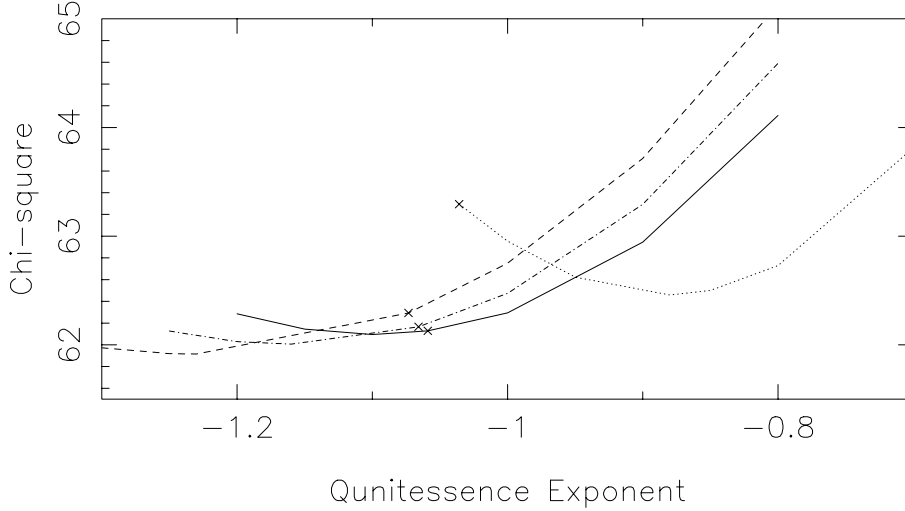


Figure 3:  $\chi^2$ -fit of models defined in (6) as a function of  $w_q$  for  $\Omega_q = 0.67$  (dashed),  $\Omega_q = 0.69$  (dash-dot),  $\Omega_q = 0.71$  (solid) and  $\Omega_q = 0.8$  (dotted). The  $\chi^2$  of equivalent quintessence models to DDMs with  $\tau = 5\tau_0$  and same  $\Omega_q$  is also shown. Except  $\Omega_q = 0.8$  model, others are all the best fit to DDM. For  $\Omega_q = 0.8$ , a stable DM fits the data better, but the fit is poorer than former models.

all these models, except  $\Omega_q = 0.8$  are compatible with data.

One has to remark that  $\Omega_q$  and  $w_q$  are not completely independent (see Footnote 2) and models with smaller  $\Omega_q$  and smaller  $w_q$  has even smaller  $\chi^2$ . In fact the best fit corresponds to  $\Omega_q = 0.5$ ,  $w_q = -2.6$  with  $\chi^2 = 61.33$ . The rejection of these models however is based on physical grounds. In fact, if the quintessence matter is a scalar field, to make such a model, not only its potential must be negative, but also its kinetic energy must be comparable to the absolute value of the potential and this is in contradiction with very slow variation of the field. In addition these models are unstable against perturbations. It is however possible to make models with  $w_q < -1$ , but they need unconventional kinetic term [16].

These results are compatible with analysis performed by Garnavich P., *et al.* [13]. However, based on null energy condition [15], they only consider models with  $w_q \geq -1$ . This condition should be satisfied by non-interacting matter and by total energy-momentum tensor. As our example of a decaying matter shows, a component or an equivalent component of energy-momentum tensor can have  $w_q < -1$  when interactions are present.

In conclusion, we have shown that a flat cosmological model including a decaying dark matter with  $\tau \sim 5\tau_0$  and a cosmological constant or a quintessence matter with  $w_q \sim -1$  at  $z < 1$  and  $\Omega_q \sim 0.7$  fits the SN-Ia data better than models with a stable DM or  $w_q > -1$ .

The effect of a decaying dark matter on the Cosmic Equation of State (CES) is a distinctive signature that can hardly be mimicked by other phenomena, e.g. conventional sources of Cosmic Rays. It is an independent mean for verifying the hypothesis of a decaying UHDM. In fact if a decaying DM affects CES significantly, it must be very heavy. Our simulation of a decaying DM with  $M \sim 10^{12} \text{eV}$  and  $\tau = 5\tau_0$  leads to an over-production by a few orders of magnitude of  $\gamma$ -ray background at  $E \sim 10^9 - 10^{11} \text{eV}$  with respect to EGRET observation [14]. Consequently, such a DM must have a lifetime much longer than  $5\tau_0$ . However, in this case it can not leave a significant effect on CES.

The only other alternative for making a quintessence term in CES with  $w_q$  slightly smaller than  $-1$ , is a scalar field with a negative potential. Nevertheless, as most of quintessence models originate from SUSY, the potential should be strictly positive. Even if a negative potential or unconventional models are not a prohibiting conditions, they rule out a large number of candidates.

Present SN-Ia data is too scarce to distinguish with high precision between various models. However, our results are encouraging and give the hope that SN-Ia observations will help to understand better the nature of dark matter in addition to cosmology of the Universe.

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